

1 (Sem-5/FYUGP) MAT 43 MJ

$$-30 - 22 - 52$$

2025

$$22 - 13 = 9$$

$$-18 - (-30)$$

$$-18 + 30$$

$$\begin{array}{r} 30 \\ 18 \\ \hline 12 \end{array}$$

MATHEMATICS

(Major)

$$\begin{array}{r} 48 \\ -35 \\ \hline 13 \end{array}$$

Paper : MAT0500304

(Numerical Analysis—I)

Full Marks : 45

Time : 2 hours

$$\begin{array}{r} 70 \\ 48 \\ \hline 22 \end{array}$$

$$40 - 70$$

$$-30$$

$$22 - 40$$

$$-8$$

$$-52 - 9$$

$$-61$$

$$12 - (-52)$$

$$12 + 52$$

$$64$$

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed :

$$1 \times 5 = 5$$

(a) In the Gaussian elimination method, the coefficient matrix of the linear system of equations reduces to

$$64 - (-61)$$

$$125$$

(i) upper triangular matrix

(ii) diagonal matrix

(iii) lower triangular matrix

(iv) None of the above

(Choose the correct option)

$$\begin{array}{r} 22 \\ 13 \\ \hline 9 \end{array}$$

$$30$$

$$-18 - (-30)$$

$$-18 + 30$$

26A/119

$$\begin{array}{r} 20 \\ 18 \\ \hline 12 \end{array}$$

$$-30 - 22$$

$$-52$$

(Turn Over)

(b) Which of the following is an iterative method?

- (i) Gauss-Jordan method
 - (ii) Lagrange's method
 - (iii) Gauss-Seidel method
 - (iv) Newton's divided difference method
- (Choose the correct option)

(c) Evaluate $\Delta^2(3e^x)$.

(d) What is numerical integration? Mention one method of numerical integration.

(e) In Simpson's $\frac{1}{3}$ rd rule, the degree of interpolating polynomial is

- (i) 1
 - (ii) 3
 - (iii) 2
 - (iv) 5
- (Choose the correct option)

2. Answer any *five* of the following questions.:

$$2 \times 5 = 10$$

(a) Prove the operator relation

$$(1 + \Delta)(1 - \nabla) \equiv 1$$

(b) Given $u_0 = 580$, $u_1 = 556$, $u_2 = 520$, $u_4 = 385$, find the value of u_3 .

(c) A second-degree polynomial passes through $(0, 1)$, $(1, 3)$, $(2, 7)$ and $(3, 13)$. Find the polynomial.

(d) Construct a divided difference table for the following data :

x	:	0	2	3	4	7	9
$f(x)$	:	4	26	58	112	466	922

(e) Consider the following system of equations :

$$4x_1 + x_2 + x_3 = 2$$

$$x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 3x_3 = -4$$

To use Jacobi iteration method

$$x^{(k+1)} = Hx^{(k)} + c$$

for solving this system, find out the coefficient matrix H .

(f) Write a short note on piecewise linear interpolation method.

(g) What is Richardson's extrapolation? Write briefly.

26-4
22
2

21

102

$$-18 - (-30) \\ -18 + 30$$

(4)

$$36 + 15 \\ \underline{51}$$

$$\begin{array}{r} 20 \\ 18 \\ \hline 12 \\ -30 - 22 \\ \hline -52 \end{array}$$

(h) Write the formulae for first- and second-order derivatives of $y = f(x)$ using Newton's forward difference interpolation formula.

(i) Write the general scheme of modified Euler's method.

(j) If $M(h) = \frac{1}{2h} [-y(x+2h) + 4y(x+h) - 3y(x)]$, show that

$$y'(x) - M(h) = c_1 h^2 + c_2 h^3 + c_3 h^4 + \dots$$

where $c_1, c_2, c_3, \dots$ are constants independent of h .

$$\begin{array}{r} 22 \\ 13 \\ \hline 9 \end{array}$$

3. Answer any four of the following questions :

$$5 \times 4 = 20$$

(a) Find the number of students from the following data who secured marks not more than 45 :

Marks	:	30-40	40-50	50-60	60-70	70-80
No. of students	:	35	48	70	40	22

(b) Show that the n th divided difference of a polynomial of degree n is constant.

$$4 - 3 \\ \underline{\quad}$$

$$\begin{array}{r} 40 \\ -70 \\ \hline -30 \end{array}$$

$$\begin{array}{r} 22 \\ 40 \\ -22 \\ \hline 18 \end{array}$$

(5)

- (c) Solve the following system of equations using Gauss elimination method :

$$2x_1 + 4x_2 + x_3 = 3$$

$$3x_1 + 2x_2 - 2x_3 = 2$$

$$x_1 - x_2 + x_3 = 6$$

- (d) Construct the Gauss-Seidel scheme for the system

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1$$

- (e) Find $f'(4)$ from the following table :

x	:	1	2	4	8	10
$f(x)$	:	0	1	5	21	27

- (f) Evaluate

$$\int_{0.5}^{0.7} \sqrt{x} e^{-x} dx$$

by Simpson's $\frac{1}{3}$ rd rule dividing the interval into four equal parts.

- (g) Calculate the approximate value of

$$\int_{-3}^3 x^4 dx$$

using trapezoidal rule dividing the interval into six equal parts. Compare with the exact value.

- (h) If $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 0$, find $y(0.3)$ using Euler's method with $h = 0.1$.

4. Answer any one of the following questions : 10

- (a) Let $y = f(x)$ be a function of the independent variable x and $y_0, y_1, \dots, y_n$ be the values of y corresponding to $x_0, x_1, \dots, x_n$. Derive Lagrange's interpolation formula to find the value of y at any point x between x_0 and x_n . Use it to find $f(4)$ from the following table :

5+5=10

x	:	0	1	2	5
$f(x)$	:	2	5	7	8

- (b) Obtain the piecewise quadratic interpolating polynomial for the function $f(x)$ defined by the following data :

x	:	-2	-1	1	2	4
$f(x)$	:	-29	-8	-2	-5	7

- (c) Evaluate $\int_1^2 e^x dx$ using trapezoidal rule considering 2, 3, 5, 9 nodes and Romberg integration method.

(7)

(d) Solve the initial value problem

$$\frac{dy}{dx} = -2xy^2, \quad y(0) = 1$$

using the mid-point method with $h=0.2$ over the interval $[0, 1]$.

$$36 - 3x \quad \left(\begin{matrix} -5 \\ \star \star \star \end{matrix} \right)$$

$$36 + 15$$

$$\frac{-24 - 3x}{-24 + 24} \quad \left(\begin{matrix} -8 \\ \star \star \star \end{matrix} \right)$$

$$4 - 3x \quad (-7) + 21$$

$$12 - (-52)$$

$$\begin{matrix} -2-4 \\ -6 \end{matrix}$$

$$2-1$$

$$\begin{matrix} 64 - (-61) \\ 64 + 61 \\ \underline{125} \end{matrix}$$

$$\begin{matrix} -52-9 \\ -61 \end{matrix}$$

$$2x \quad (-8)$$